

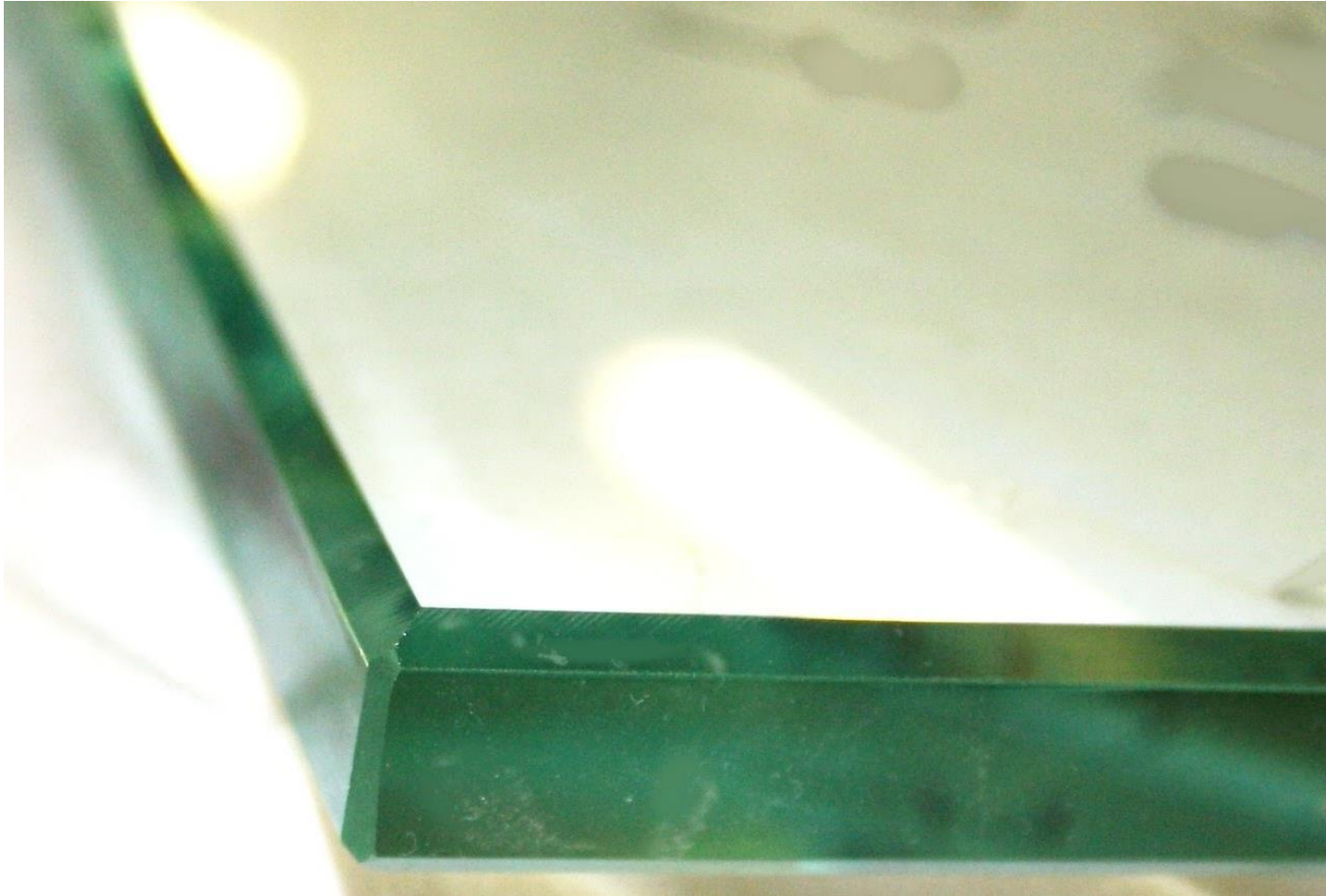
# Advanced 3D graphics for movies and games (NPGR010)

## – Volume Rendering

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Slides by Pascal Grittmann, Jaroslav Křivánek

# Overview



- So far:
  - Light interactions with surfaces
  - Assume vacuum in and around objects
- This lecture:
  - Participating media
  - How to represent volumetric data
  - How to compute volumetric lighting effects
  - How to implement a very basic volume renderer

# Fog & clouds

An aerial photograph of a dense forest during sunrise or sunset. The sun is low on the horizon, creating a strong glow and casting long, golden rays of light through the trees. The ground is covered in a thick layer of fog or low clouds, which catches the light and creates a hazy, ethereal atmosphere. The trees are mostly tall and thin, with their tops silhouetted against the bright sky. The overall color palette is dominated by warm, golden-brown and green tones.

# Underwater



source: [dailypictures.info](http://dailypictures.info)

# Surface or volume?



source: <sup>5</sup>Flickr

source: Studio Lernert & Sander





**Avatar.** Copyright © 2009 20th Century Fox



**Arrival.** Copyright © 2016 Paramount Pictures



**Big Hero 6.** Copyright © 2014 Walt Disney Enterprises, Inc.



**Mortal Engines.** Copyright © 2018 Universal Studios

# Fundamentals



# Volumetric Effects

- Light interacts not only with surfaces but everywhere inside!
- Volumes scatter, emit, or absorb light



<http://coclouds.com>



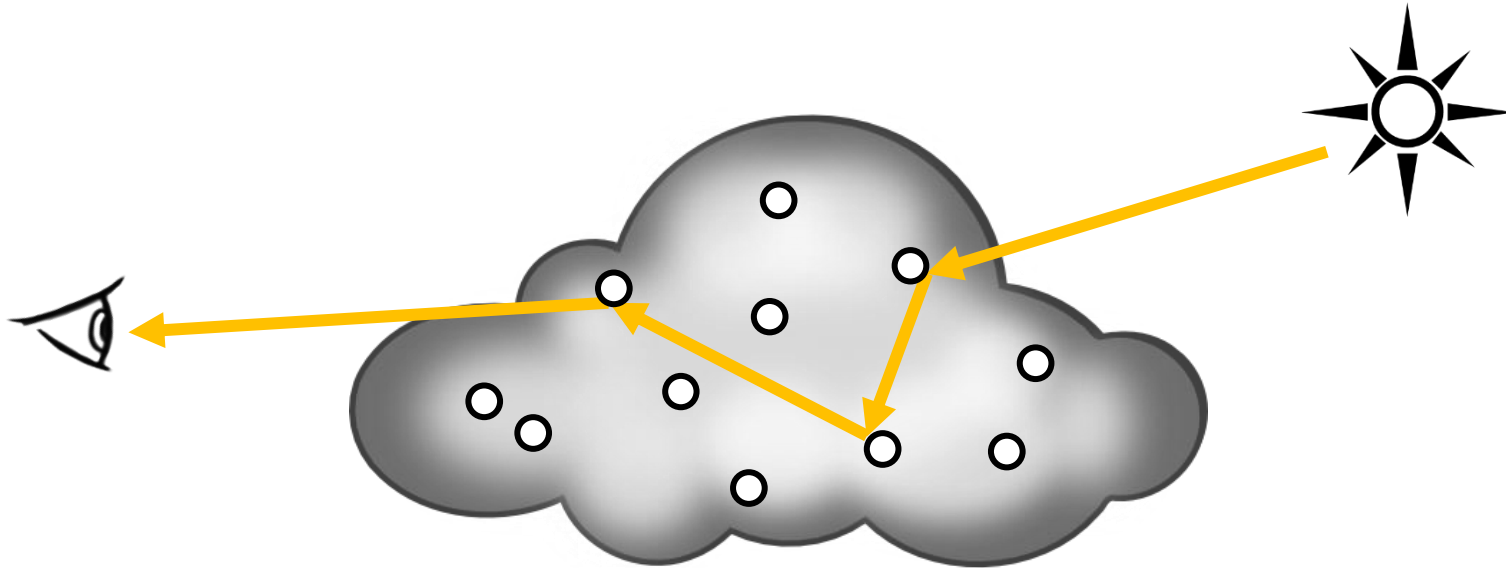
<http://wikipedia.org>



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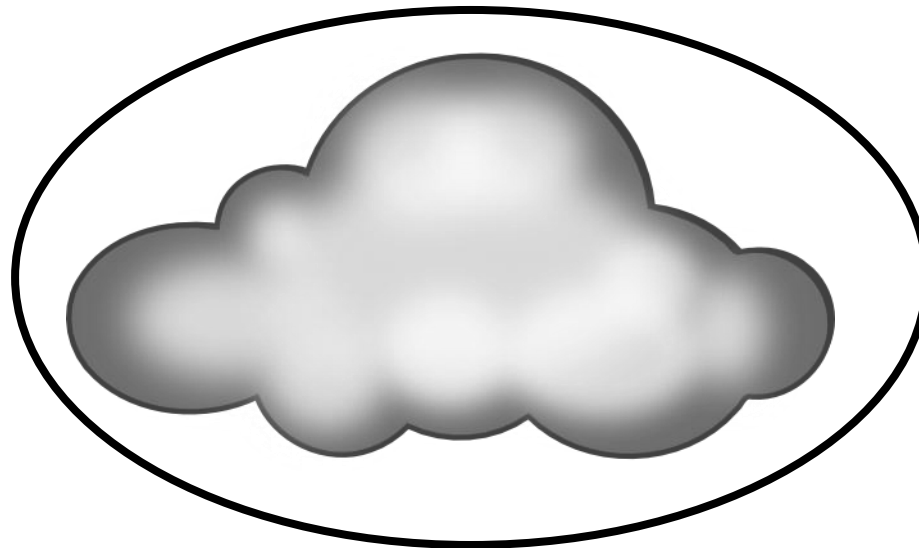
# Approximation: Model Particle Density

- Modeling individual particles of a volume is, of course, not practical
- Instead, represent statistically using the average density
- (Same idea as, e.g., microfacet BSDFs)



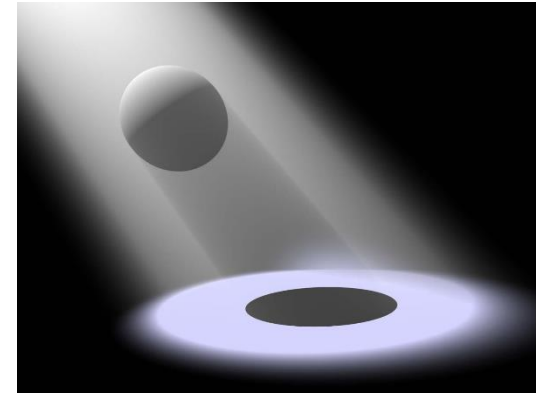
# Volume Representation

- Many possibilities (particles, voxel octrees, procedural,...)
- A common approach: Scene objects can “contain” a volume



# Volume Representation

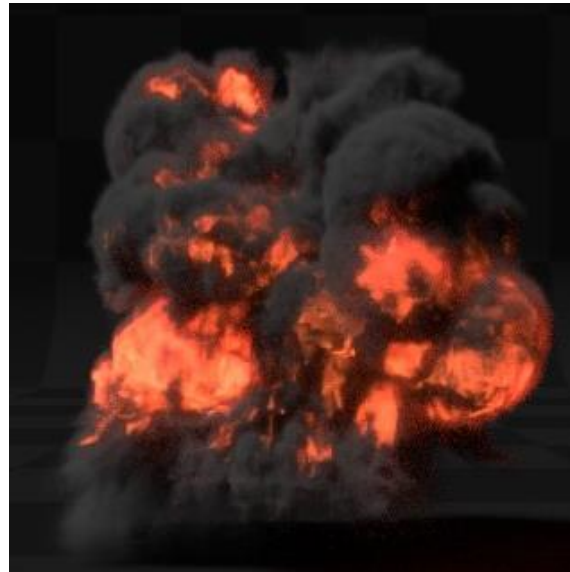
- Homogeneous:
  - Constant density
  - Constant absorption, scattering, emission,
  - Constant phase function (later)
  
- Heterogeneous:
  - Coefficients and/or phase function vary across the volume
  - Can be represented using **3D textures**
  - (e.g., voxel grid, procedural)



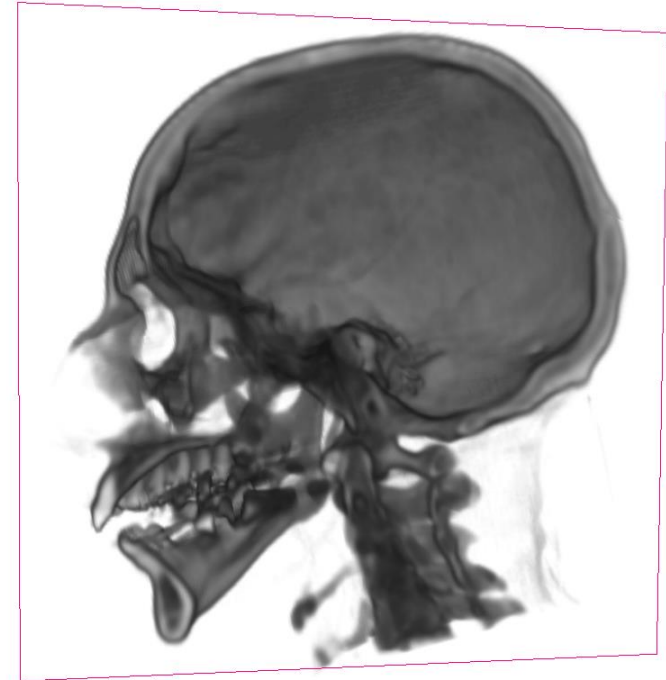
<http://wikipedia.org>

# Data Acquisition

- Real-world measurements via tomography
- Simulation, e.g.,
  - Fluids, (<https://www.youtube.com/watch?v=Bljj3Qcmbf4>)
  - Fire and smoke,
  - Fog



<https://docs.blender.org>



# Simulating Volumes

Mathematical Formulation of Volumetric Light Transport

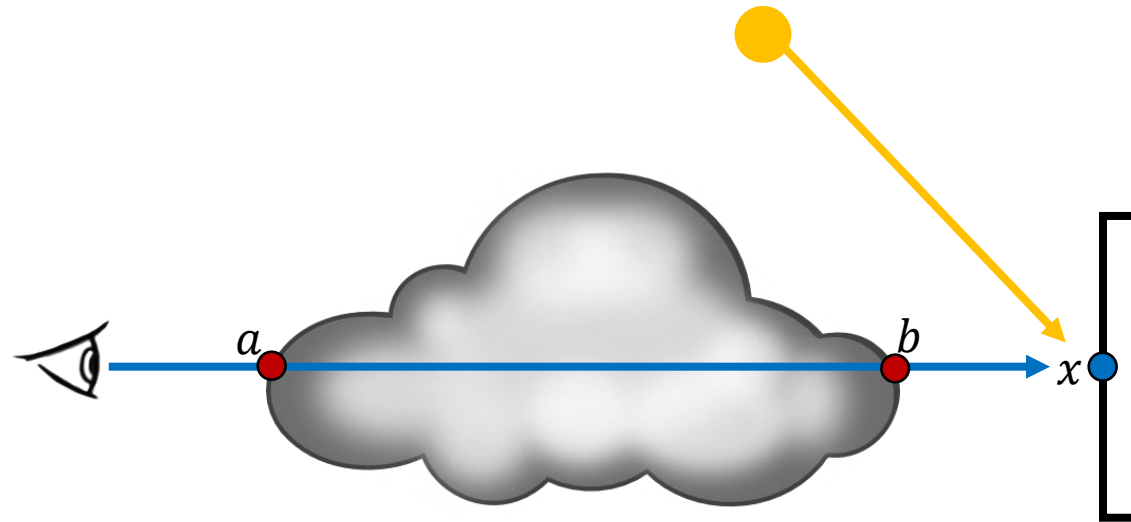
# So far: Assume Vacuum

- Compute  $L_o(x, \omega_o)$  using the rendering equation



# Volume Absorbs and Scatters Light

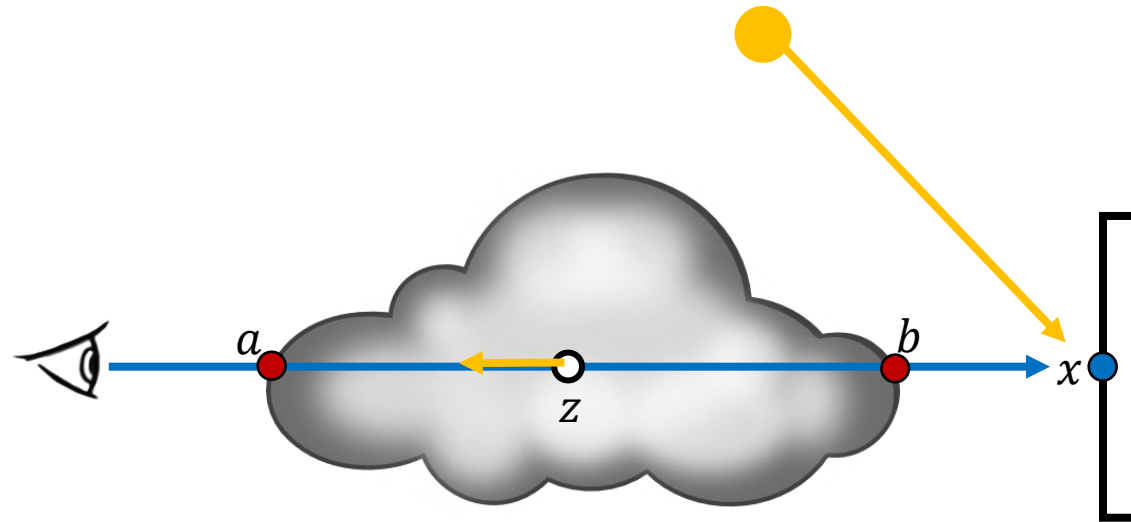
- Compute  $L_o(x, \omega_o)$  using the rendering equation
- Only a fraction  $T(a, b)L_o(x, \omega_o)$  arrives at the eye





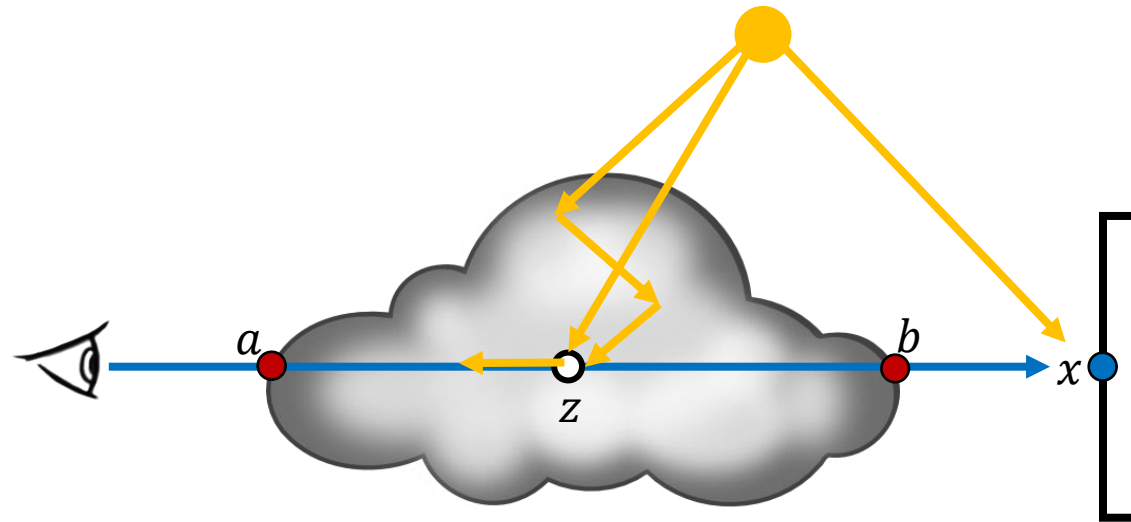
# Volume Emits Light

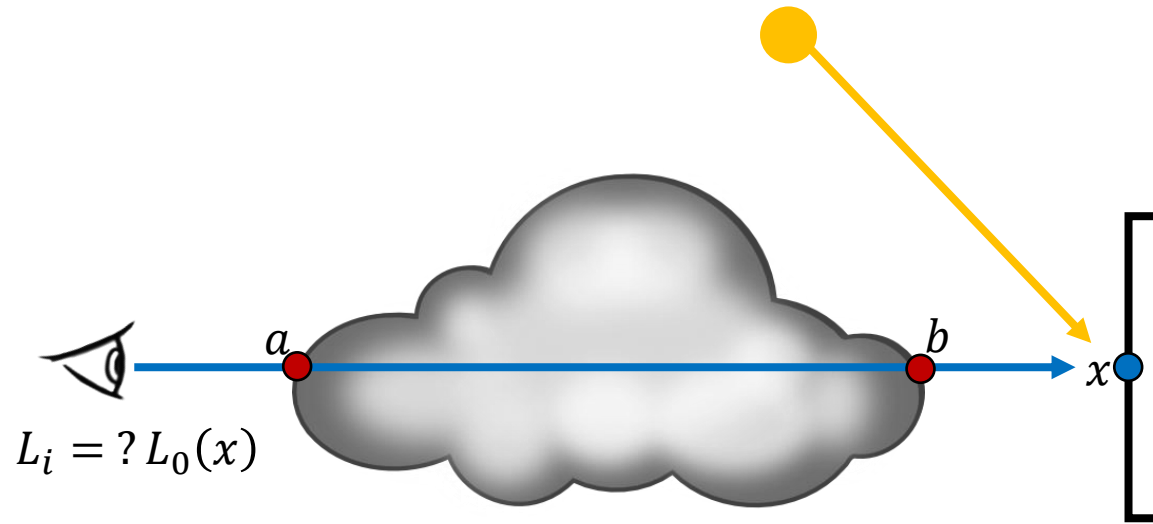
- Compute  $L_o(x, \omega_o)$  using the rendering equation
- Only a fraction  $T(a, b)L_o(x, \omega_o)$  arrives at the eye
- Every point  $z$  between  $a$  and  $b$  might emit light



# Volume Scatters Light

- Compute  $L_o(x, \omega_o)$  using the rendering equation
- Only a fraction  $T(a, b)L_o(x, \omega_o)$  arrives at the eye
- Every point  $z$  between  $a$  and  $b$  might emit light
- Every point  $z$  might be illuminated through the volume





# Attenuation

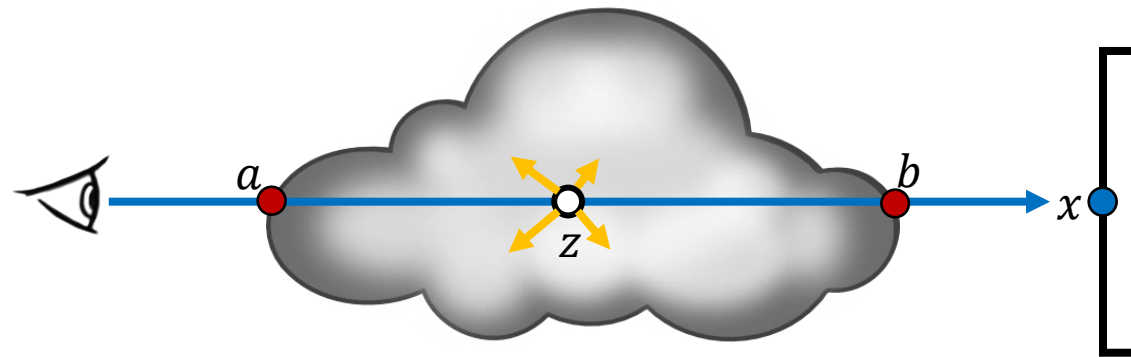
Computing Absorption and Out-Scattering



<http://commons.wikimedia.org>

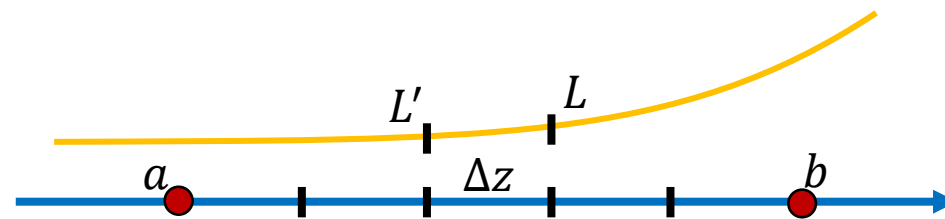
# Attenuation = Absorption + Out-Scattering

- Every point in the volume might absorb light or scatter it in other directions
- Modeled by absorption and scattering coefficients:  $\mu_a(z)$  and  $\mu_s(z)$  (both in  $[m^{-1}]$ )
- Might depend on position, direction, time, wavelength,...
- For simplicity: we assume only positional dependence



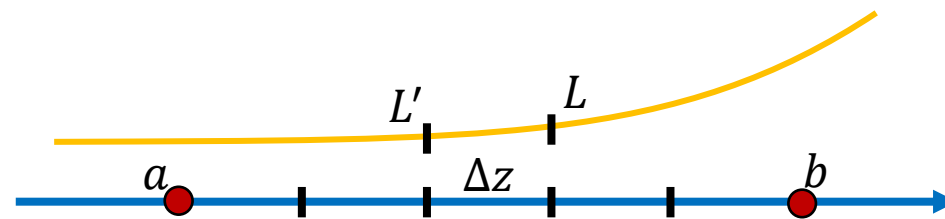
# Computing Absorption – Intuition

- Consider a small segment  $\Delta z$
- Along that segment, radiance is reduced from  $L$  to  $L'$
- $L' = L - L(\mu_a \Delta z)$
- Where  $\mu_a$  is the percentage of radiance that is absorbed (per unit distance)



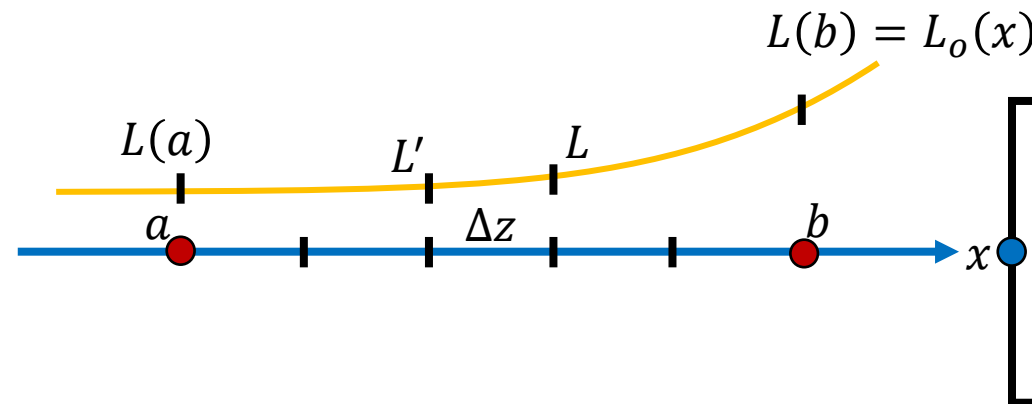
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- $L' = L - L(\mu_a \Delta z)$
- Where  $\mu_a$  is the percentage of radiance that is absorbed (per unit distance)
  
- Lets rewrite this:
- $\Delta L = L' - L = -L\mu_a \Delta z$



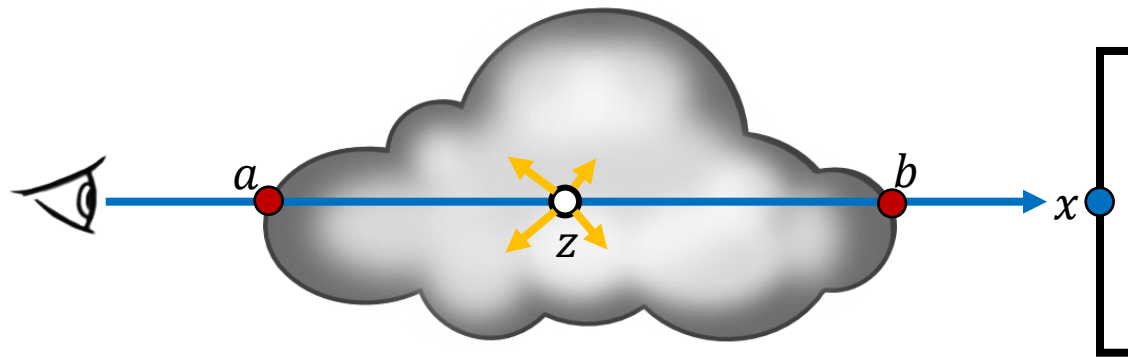
# Computing Absorption – Exponential Decay

- $\Delta L = -\mu_a L \Delta z$
- For infinitely small  $\Delta z$ , this becomes
- $dL = -\mu_a L dz$
- A differential equation that models exponential decay!
- Solution:  $L(a) = L_o(x) e^{-\int_0^a \mu_a(t) dt}$



# Computing Out-Scattering

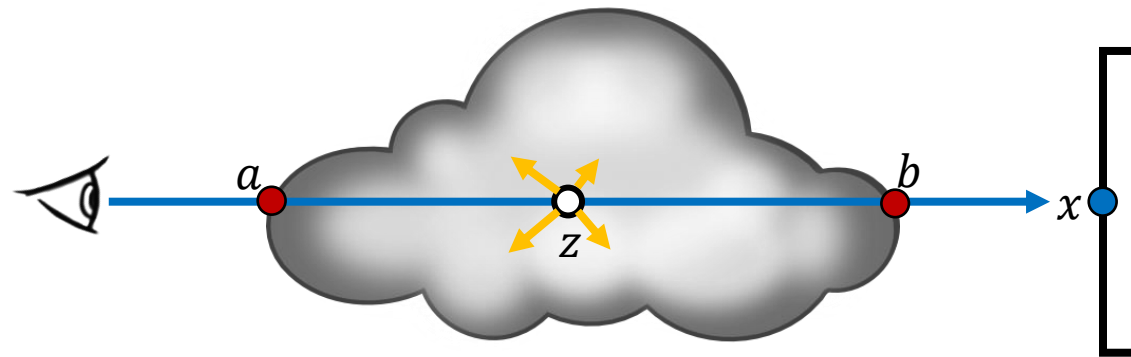
- Same as absorption, only different factor!
- $L(a) = L_o(x) e^{-\int_0^a \mu_s(t) dt}$





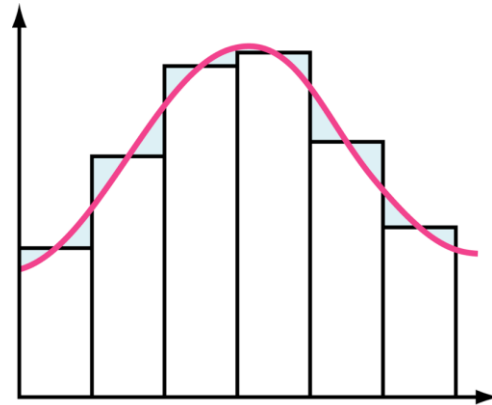
# Computing Attenuation

- Fraction of light that is either absorbed or out-scattered (per unit distance)
- $\mu_t = \mu_a + \mu_s$
- Many different names: extinction / attenuation / transport coefficient
- $L(a) = L_o(x) e^{-\int_0^a (\mu_a(t) + \mu_s(t)) dt}$
- Attenuation:  $T(a, b) = e^{-\int_b^a \mu_t(t) dt}$



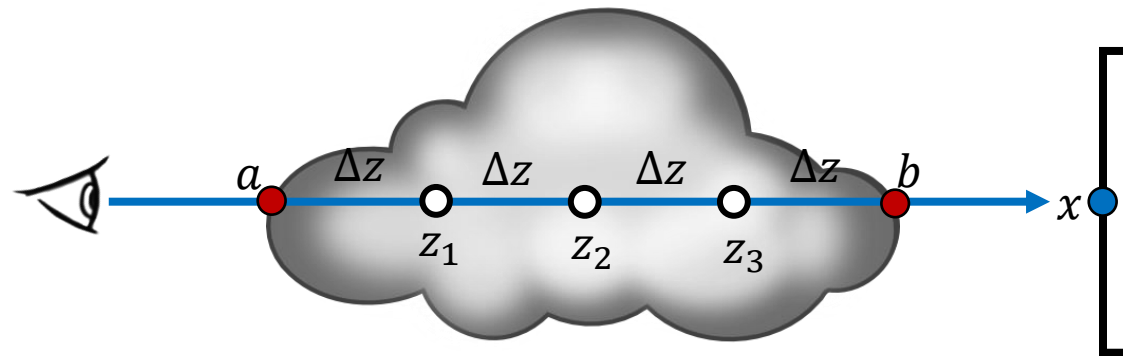
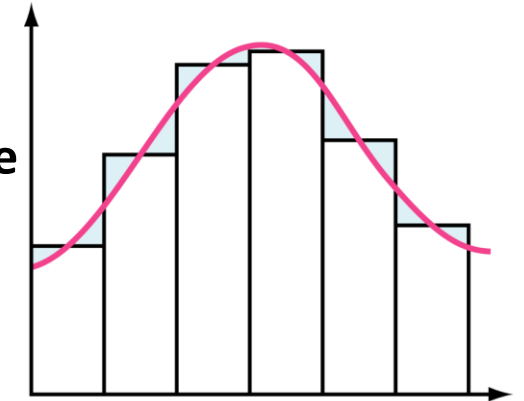
# Estimating Attenuation

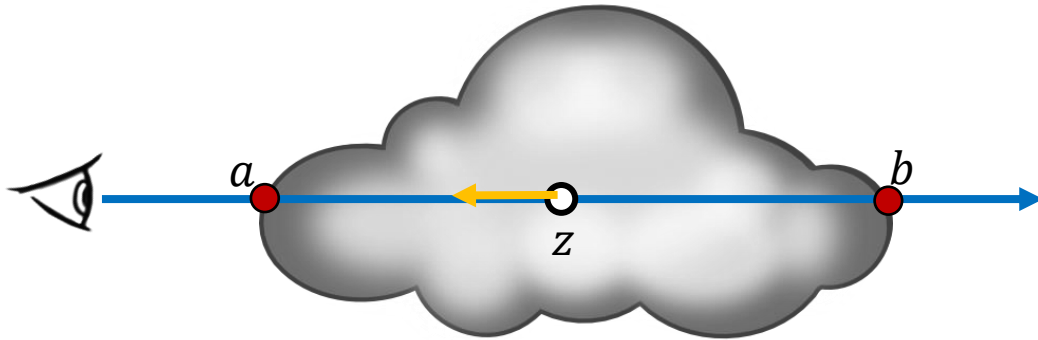
- We need to evaluate another integral:
  - $T(a, b) = e^{-\int_b^a \mu_t(t) dt}$
- Many approaches, e.g., ~~Monte Carlo integration~~ or **deterministic quadrature**



# Estimating Attenuation – Ray Marching

- We need to evaluate another integral:
  - $T(a, b) = e^{-\int_b^a \mu_t(t) dt}$
- Many approaches, e.g., ~~Monte Carlo integration~~ or **deterministic quadrature**
- Ray marching: evaluate at discrete positions (fixed stepsize  $\Delta z$ )
- $\int_b^a \mu_t(t) dt \approx \sum_i \mu_t(z_i + \varepsilon) \Delta z$
- Randomized **offset**  $\varepsilon$  for each ray to avoid aliasing problems





# Emission

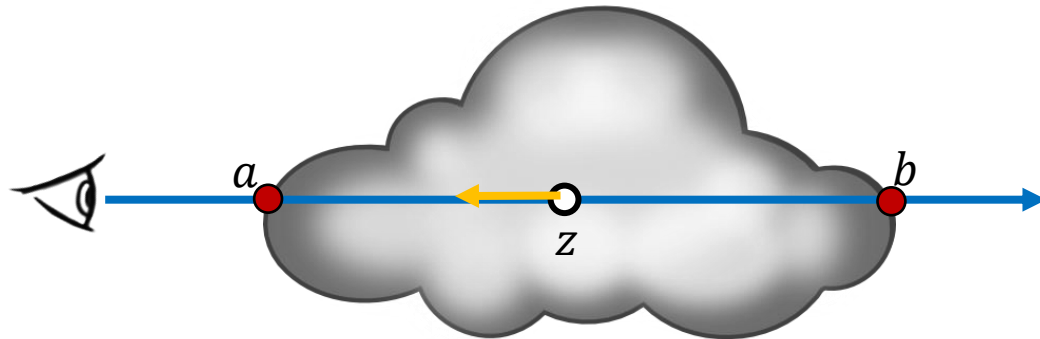
Explosions!



<http://wikipedia.org>

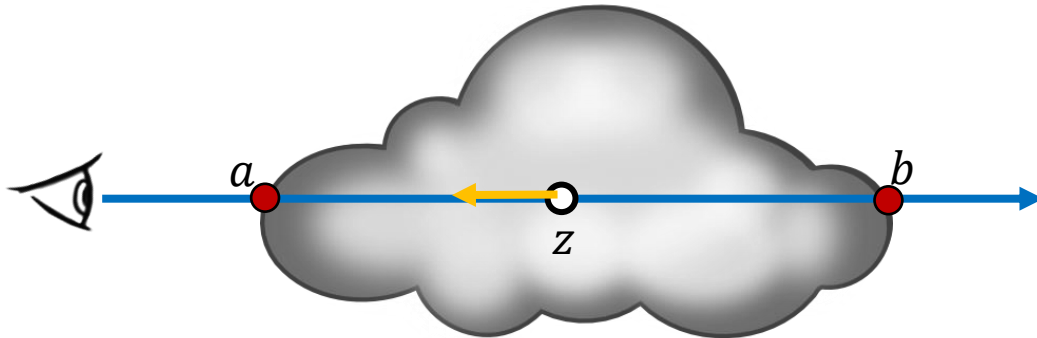
# Every Point Might Emit Light

- Assume  $z$  emits  $L_e(z)$  towards  $a$
- Some of that light might be absorbed or out-scattered: It is attenuated
- $L(a) = L_e(z) T(z, a)$



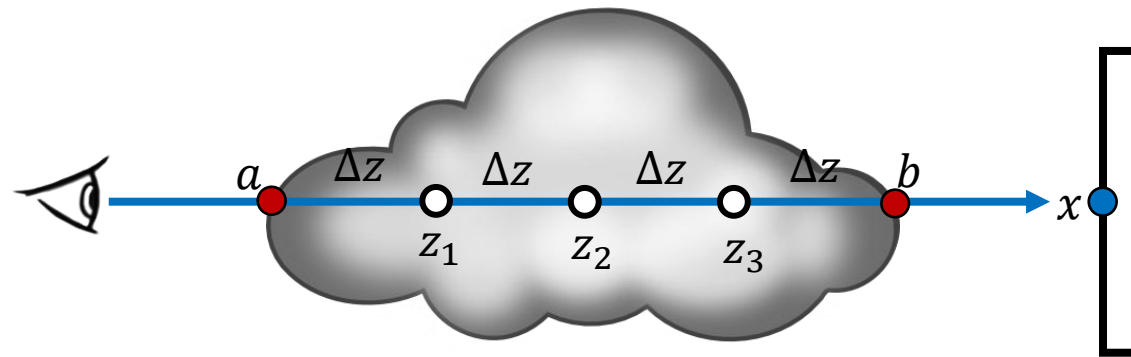
# Every Point Might Emit Light

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- Some of that light might be absorbed or out-scattered: It is attenuated
- $L(a) = L_e(z) T(z, a)$
- Happens at every point along the ray!
- $L(a) = \int_a^b L_e(z) T(z, a) dz$
- Another integral...



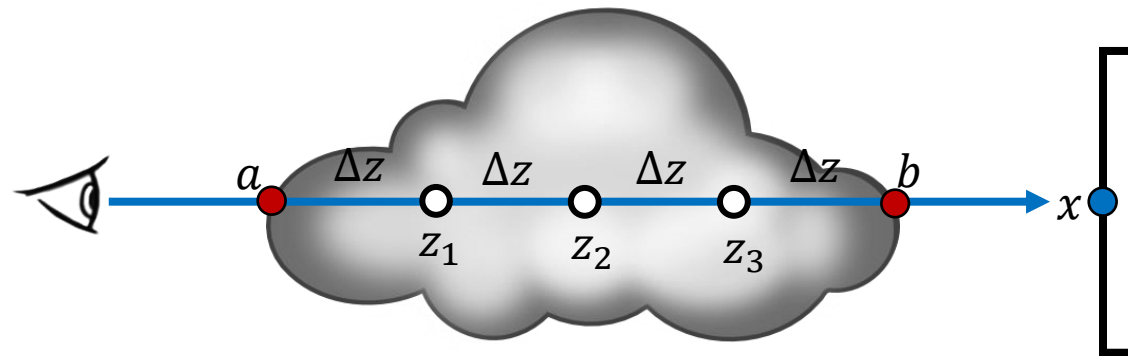
# Ray Marching for Emission

- Same as before: integrate via quadrature
- $\int_a^b L_e(z) T(z, a) dz \approx \sum_i L_e(z_i) T(z_i, a) \Delta z$
- Attenuation  $T(z_i, a)$  estimated as before

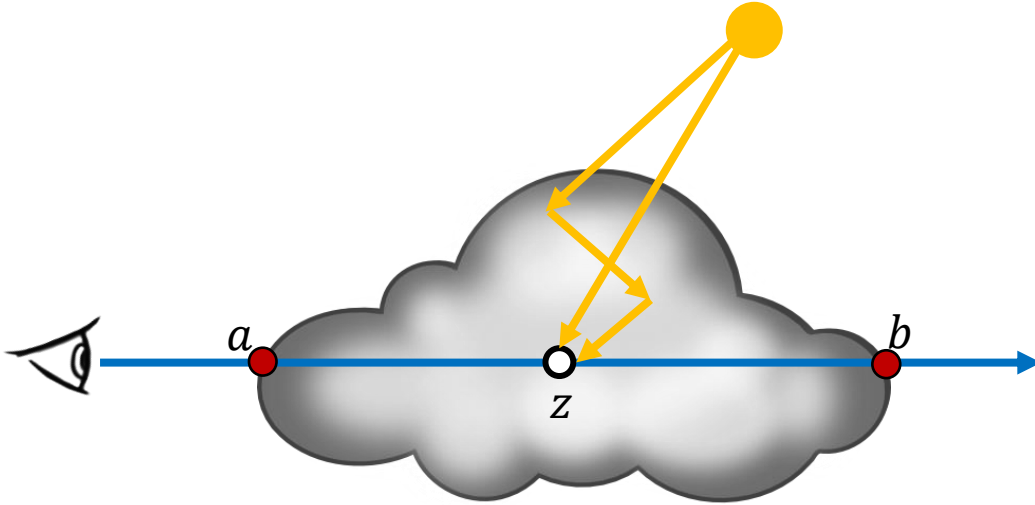


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- Attenuation  $T(z_i, a)$  estimated as before
- Attenuation can be incrementally updated:
  - $T(z_i, a) = T(z_{i-1}, a) T(z_i, z_{i-1})$
  - (because it is an exponential function)







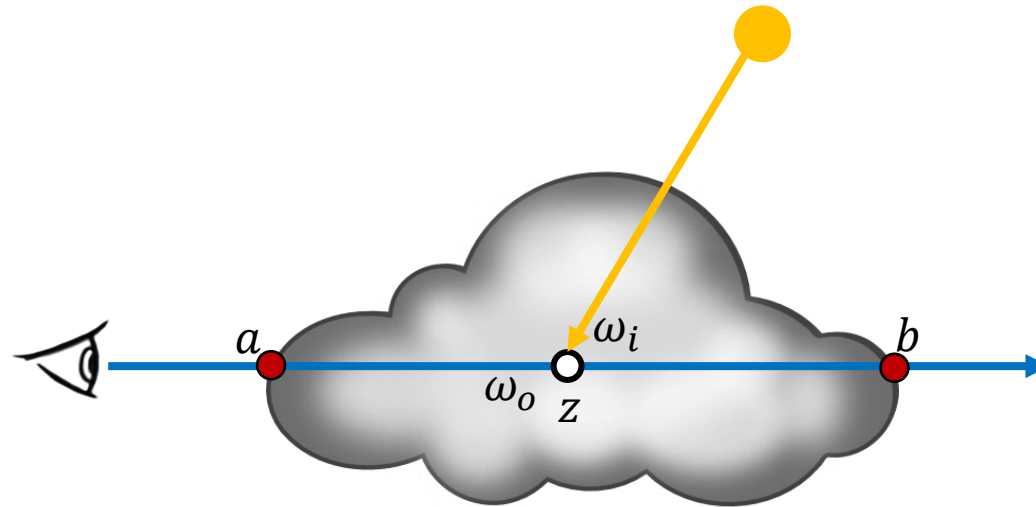
# In-Scattering

Accounting for “Reflections” Inside the Volume



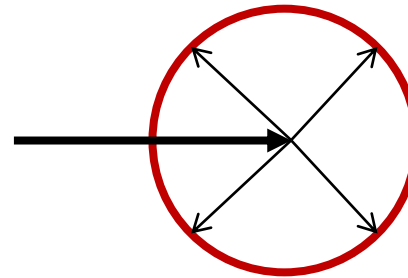
# Direct Illumination (Single-scattering)

- Account for the (attenuated) direct illumination at every point  $z$
- Similar to the rendering equation:
- $L_o(z, \omega_o) = \int_{\Omega} L_i(z, \omega_i) f_p(\omega_i, \omega_o) d\omega_i$
- Integration over the whole sphere  $\Omega$
- The **phase function**  $f_p$  takes on the role of the BSDF



# Phase Functions

- $f_p(\omega_i, \omega_o)$
- Describe what fraction of light is reflected from  $\omega_i$  to  $\omega_o$
- Similar to BSDF for surface scattering
- Simplest example: isotropic phase function
  - $f_p(\omega_i, \omega_o) = \frac{1}{4\pi}$
  - (energy conservation:  $\int_{\Omega} \frac{1}{4\pi} d\omega = 1$ )



# Phase Functions: Henyey-Greenstein

- Widely used
- Easy to fit to measured data
- $$f_p(\omega_i, \omega_o) = \frac{1}{4\pi} \frac{1-g^2}{(1+g^2+2g \cos(\omega_i, \omega_o))^{\frac{3}{2}}}$$
- $g$ : asymmetry, mean cosine (scalar)
- $\cos(\omega_i, \omega_o)$ : cosine of angle between the directions

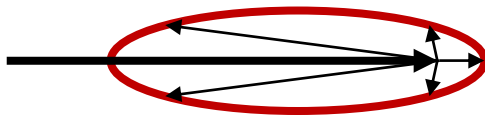
# Henyey-Greenstein: Asymmetry Parameter

- $g = 0$ : isotropic
- Negative  $g$ : back scattering
- Positive  $g$ : forward scattering

Back Scattering



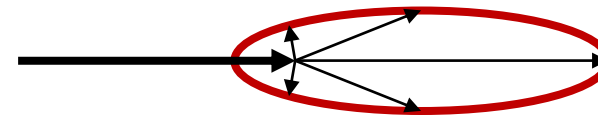
<http://commons.wikimedia.org>



Forward Scattering

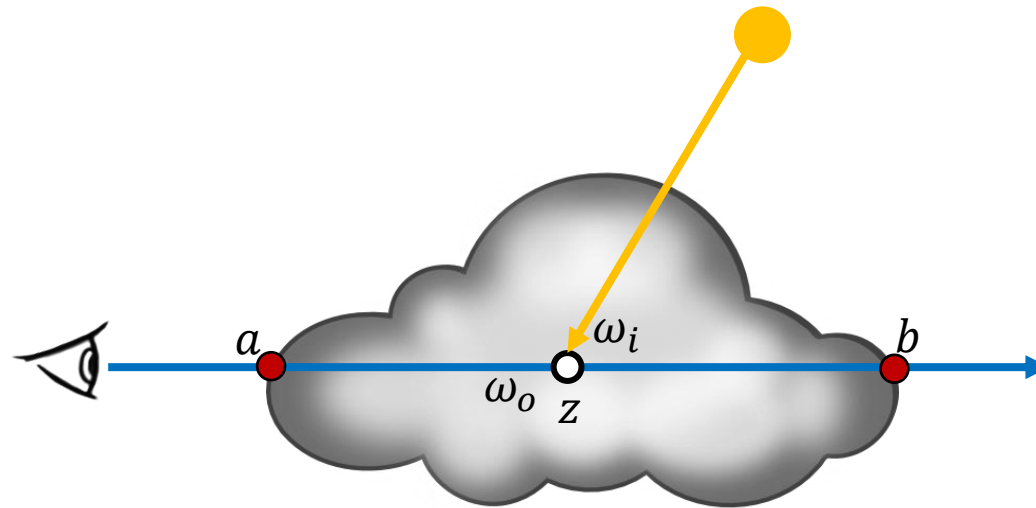


<http://coclouds.com>



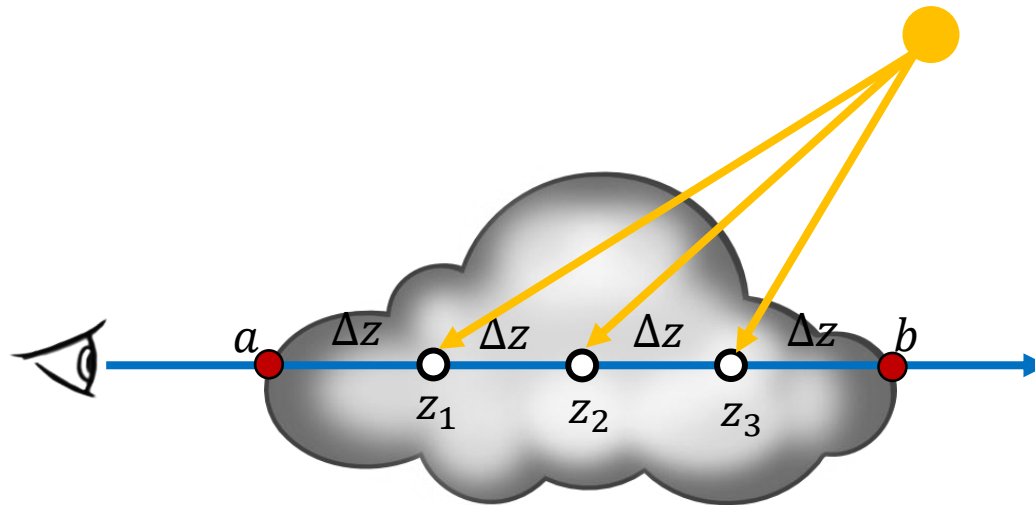
# How to Estimate Volumetric Direct Illumination

- Reflected radiance at a point  $z$ :  $L_o(z, \omega_o) = \int_{\Omega} L_i(x, \omega_i) f_p(\omega_i, \omega_o) d\omega_i$
- In our framework:
  - Sum over all lights – choose a point for each stochastically (NEE as for surfaces)
  - Trace shadow ray (as for surfaces)
  - Estimate attenuation along the shadow ray (as for surfaces)



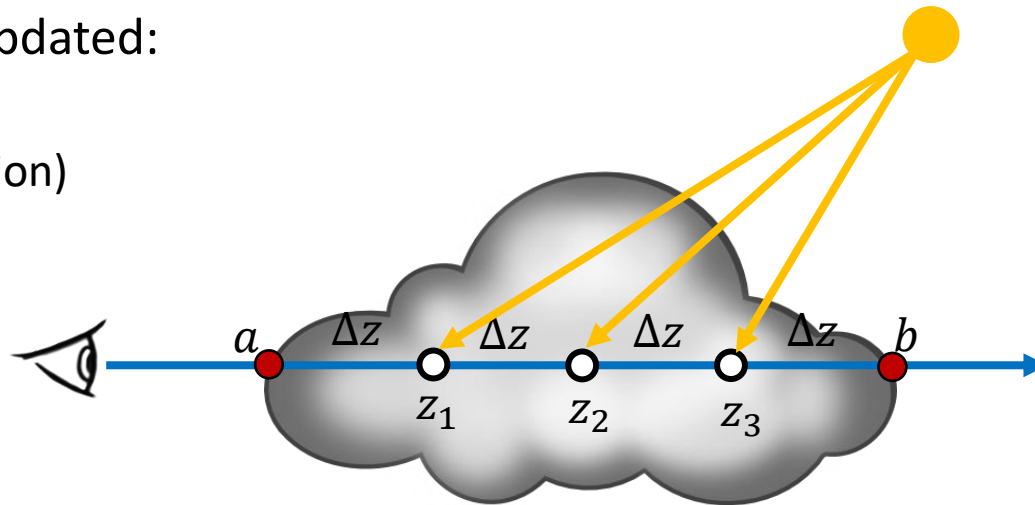
# Ray Marching to Compute In-Scattering

- Same as for emission
- Goal: estimate the integral  $\int_a^b T(z, a) L_i(z) f_p dz$
- Quadrature:
  - $\int_a^b T(z, a) L_i(z) f_p dz \approx \sum_i T(z_i, a) L_i(z_i) f_p \Delta z$



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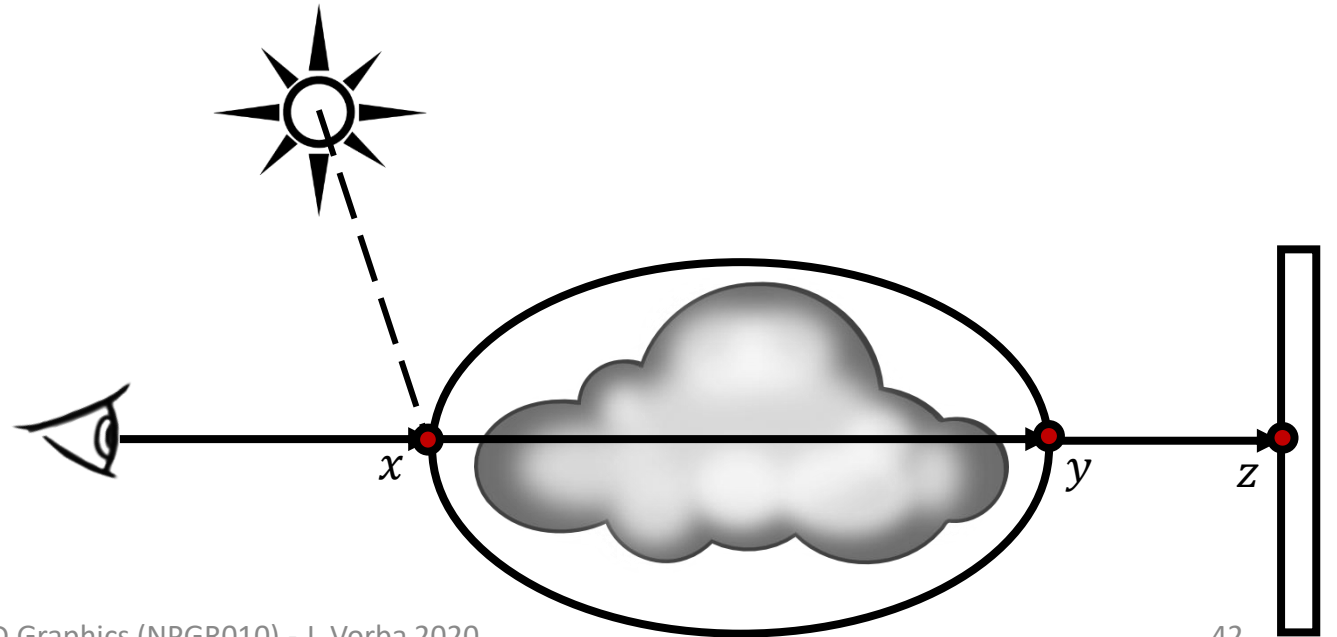


# Putting it all Together

A Simple Volume Integrator

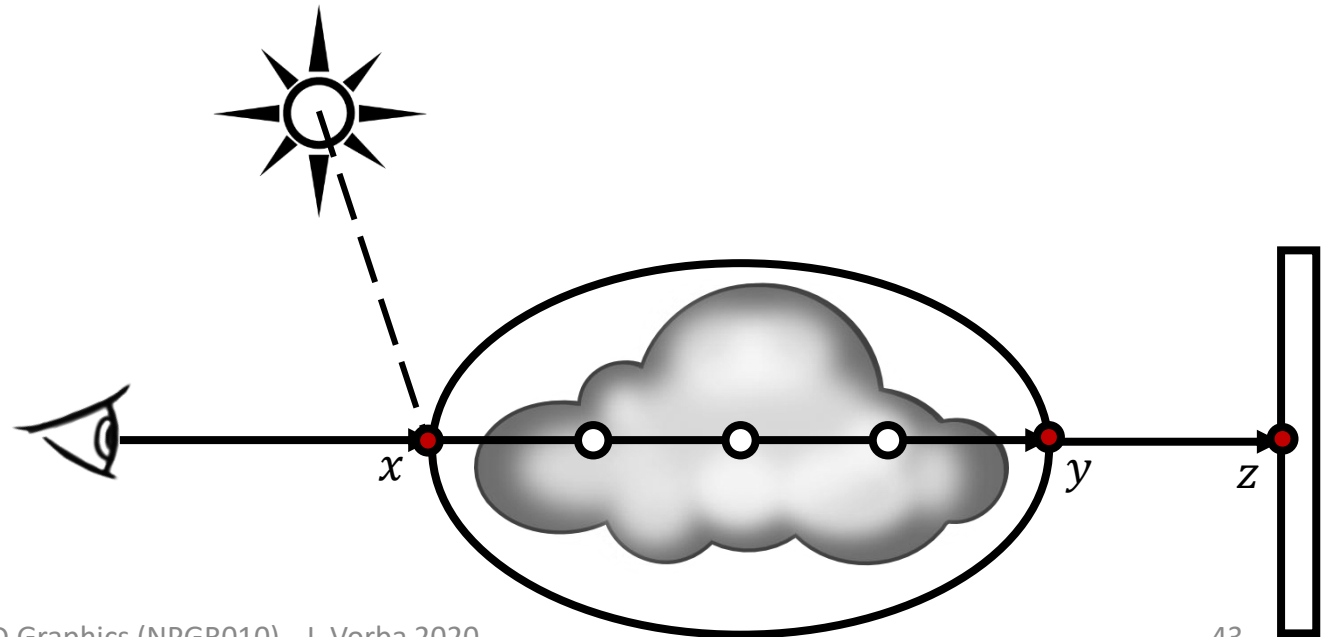
# A Simple Volume Integrator

- Estimate direct illumination at  $x$  (as before)
- If volume: continue straight ahead until no volume (yields intersections  $y, z$ )



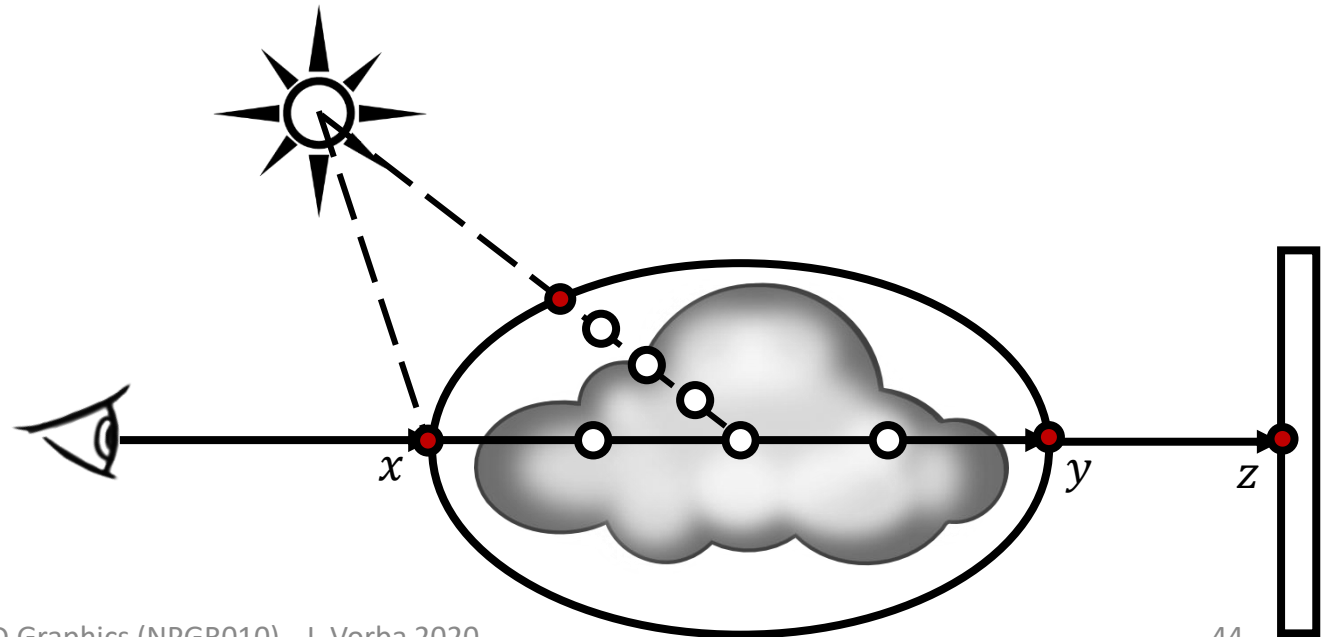
# A Simple Volume Integrator

- Estimate direct illumination at  $x$  (as before)
- If volume: continue straight ahead until no volume (yields intersections  $y, z$ )
- Ray marching to estimate attenuation, emission,



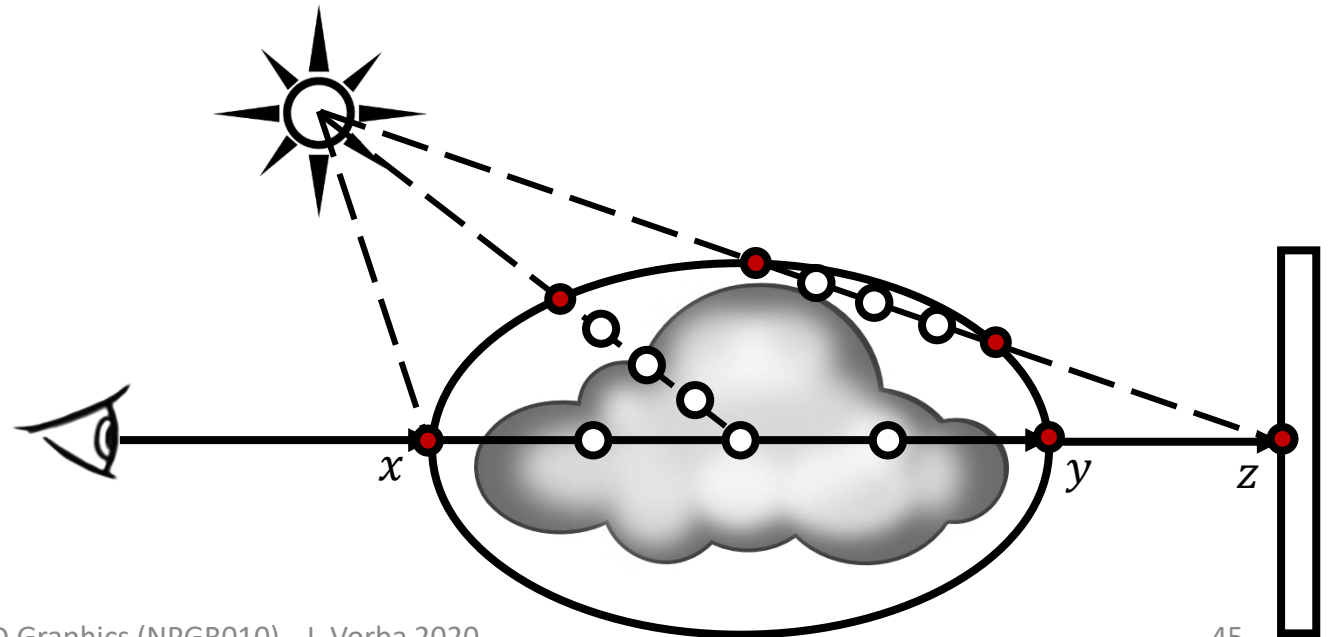
# A Simple Volume Integrator

- Estimate direct illumination at  $x$  (as before)
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  - Shadow rays to the lights + ray marching to compute attenuation



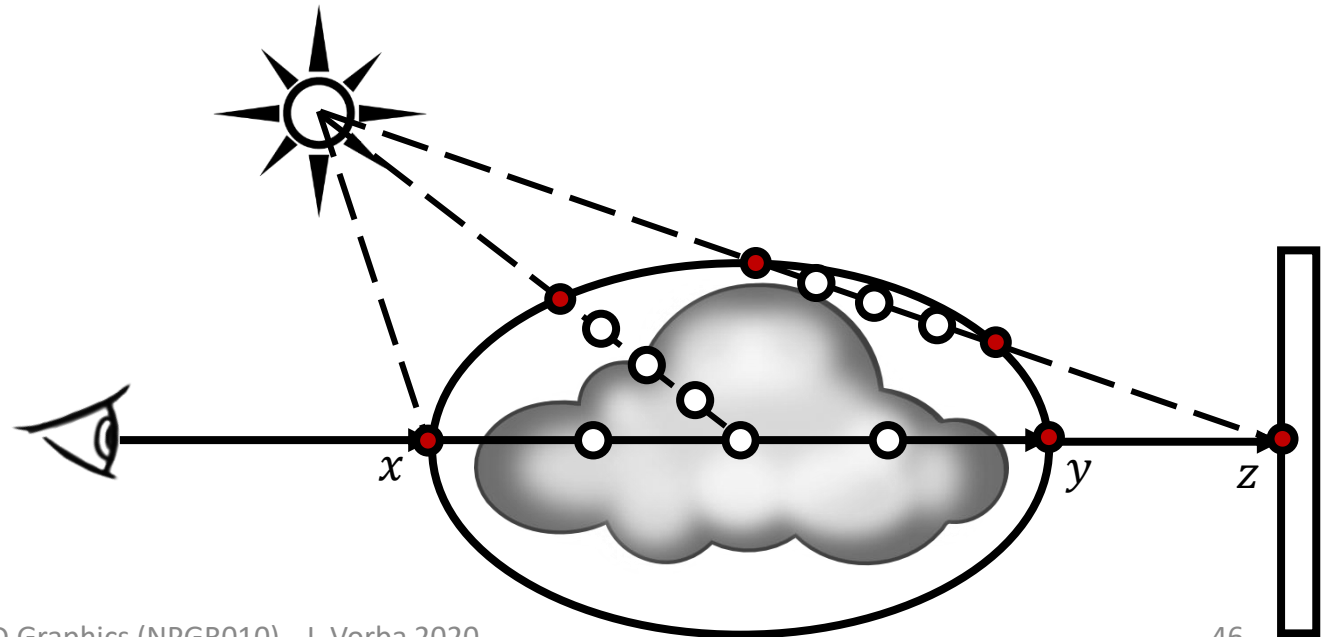
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- Estimate direct illumination at  $x$  (as before)
- If volume: continue straight ahead until no volume (yields intersections  $y, z$ )
- Ray marching to estimate attenuation, emission, and in-scattering
  - Shadow rays to the lights + ray marching to compute attenuation
- Compute illumination at  $z$  (as before)
- Add together:
  - Attenuated illumination from  $z$
  - Volumetric emission along  $\overline{xy}$
  - In-scattering along  $\overline{xy}$
  - Direct illumination at  $x$



# What next

- Quadrature (ray-marching) not suitable for multiple-scattering
  - Curse of dimensionality
- Volumetric rendering equation
- Stochastic free-flight sampling
- MIS for direct illumination
- IOR
- Overlapping volumes
- Wavelength dependency (problem even in RGB renderer)
  - Hero-wavelength sampling - Wilkie et al. [2014]